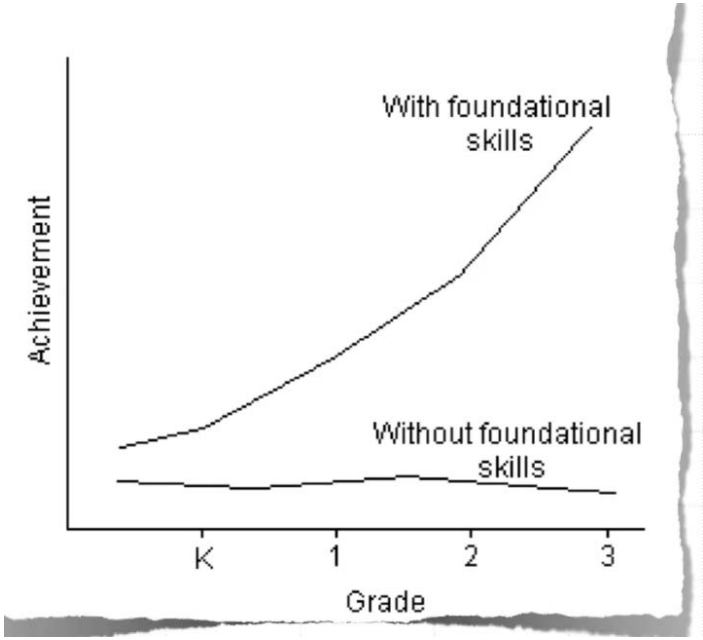
Maths in the EYFS

What are the priorities for our youngest learners?

Sue Gifford

Simon Lewis

Early maths matters: those who start behind stay behind- and the gap widens in primary school EEF, 2020



Early Intervention Foundation, 2018:

Children's understanding of number during preschool is consistently associated with their mathematical achievement in primary and secondary school.

The ages of 3 to 5 are therefore considered an ideal time to rectify income-related learning gaps in children's understanding of numbers.

Mathematical achievement in turn is consistently found to be the strongest predictor of children's overall school achievement and their success in entering the workforce.

What research tells us:
Mathematical predictors

- parents' education and home learning EIF, 2018
- self-belief OECD, 2012
- mathematical reasoning Nunes & Bryant, 2012

- **number sense** Nunes & Bryant, 2009, EIF, 2018
- **patterning** Rittle-Johnson et al, 2016
- **spatial reasoning** Hawes & Ansari, 2020

Simon Lewis

Statutory Mathematics Educational Programme

Developing a strong grounding in number is essential so that all children develop the necessary building blocks to excel mathematically. Children should be able to count confidently, develop ***a deep understanding of the numbers to 10, the relationships between them and the patterns within those numbers****. By providing frequent and varied opportunities to build and apply this understanding - such as using manipulatives, including small pebbles and tens frames for organising counting - children will develop a secure base of knowledge and vocabulary from which mastery of mathematics is built.*

In addition, it is important that the curriculum includes rich opportunities for children to develop their ***spatial reasoning skills*** *across all areas of mathematics including shape, space and measures. It is important that children* ***develop positive attitudes and interests*** *in mathematics,* ***look for patterns*** *and relationships, spot connections, 'have a go', talk to adults and peers about what they notice and not be afraid to make mistakes.*

EYFS Statutory Framework 2021

Early Learning Goals 2021

Removed: *Shape, space and measures* and problem solving

Number (in red: no research evidence)

Children at the expected level of development will:

- Have a deep understanding of number to 10, including the composition of each number;
- Subitise (recognise quantities without counting) up to 5;
- Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.

EYFS Framework 2021

Numerical Patterns ELG

Children at the expected level of development will:

- Verbally count beyond 20, recognising the pattern of the counting system;
- Compare sets of objects up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity;
- Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.

Number sense: *a feeling for numbers*

Cardinality and Counting

Understanding that the cardinal value of a number refers to the quantity, or 'howmanyness' of things it represents

Comparison

Understanding that comparing numbers involves knowing which numbers are worth more or less than each other

Composition

Understanding that one number can be made up from (composed from) two or more smaller numbers

NCETM

Counting - sequence and synchronisity

Cardinality -'*how manyness*'

Comparison – order and relative size

Composition - *numbers made up of other numbers*

Give me 9

Number predictors for 5 year olds

- **counting out** a number from a larger group (EIF, 2018)

- understanding **cardinal** and **ordinal numerals**

(Schneider et al, 2017)

Cardinal 3 - an amount

How many is 3?

Where does 3 go?

Ordinal 3 - a position

Counting

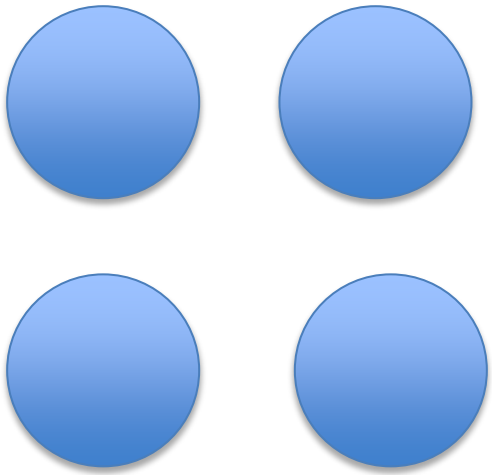
It can take 4 years to learn
to count to 20

number sequence - teen numbers: 13,14,15
- crossing boundaries 29/30

tagging 1 - 1 - co-ordinating saying & pointing

cardinal principle - last number is ‘how many’

How to teach cardinal counting



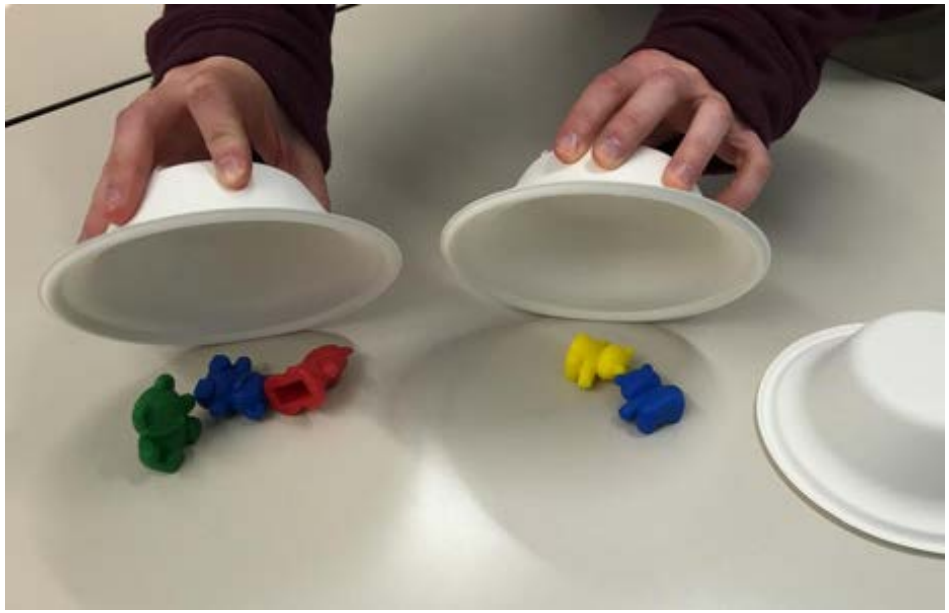
1, 2, 3, 4.

There are 4 buttons.

Subitising up to 4 helps children to understand cardinal counting

Paliwal & Baroody (2018, 2020)
Cahoon et al. 2021

Subitising



Learning Trajectories https://learningtrajectories.org/

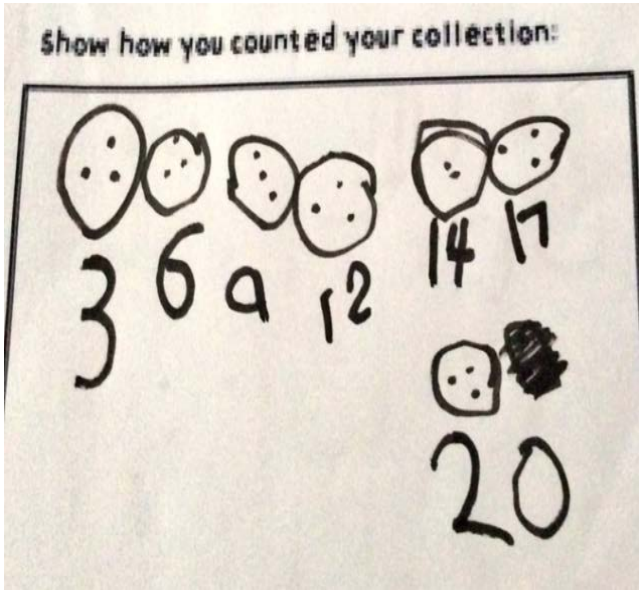
Number talk images ntimages.weebly.com

Counting collections

Gripton & Pawluch 2021

Choose:

- what
- how
- record

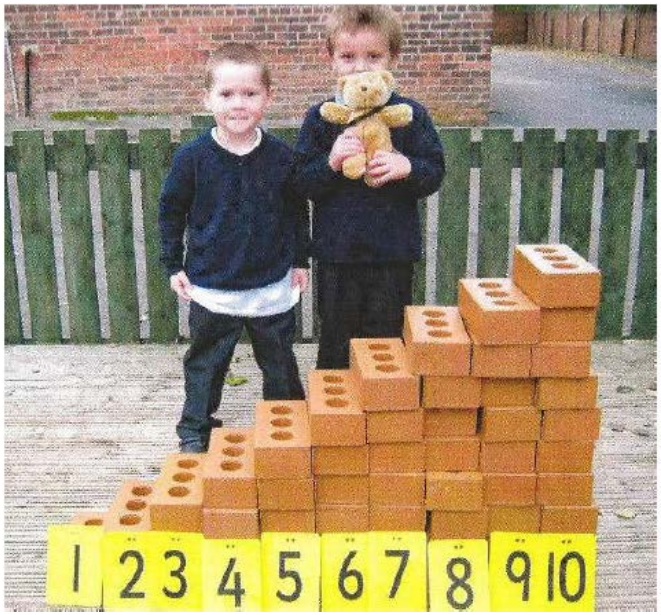


Assessing object counting

Can you give teddy 5 buttons?

ECMG ChIPs informal assessments
https://earlymaths.org/chips/

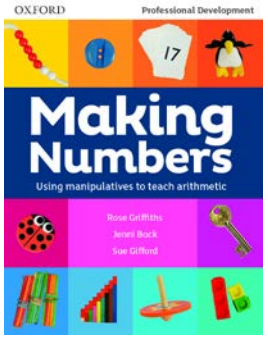
Numerical patterns

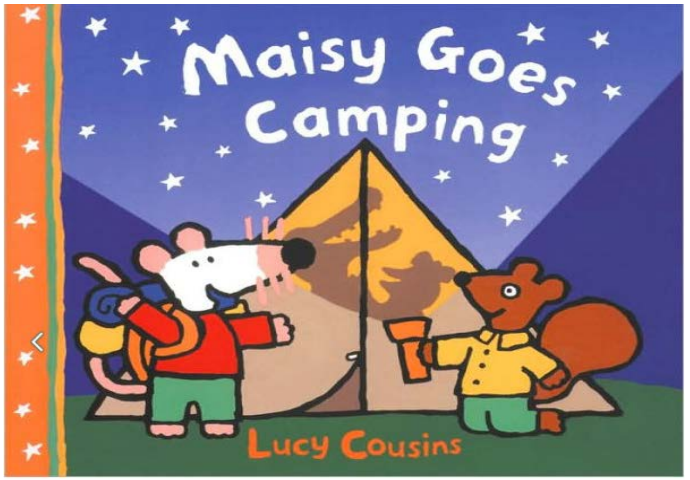


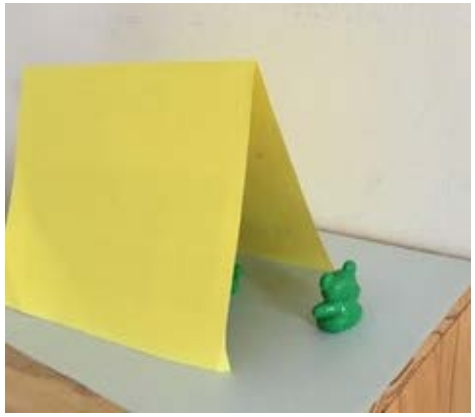
Automatic recall: not abstract number bonds

5 friends in the tent

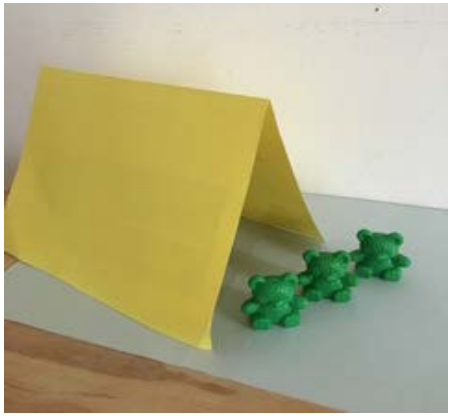
5 friends counting (animation)
Griffiths, Back & Gifford (2016*) Making numbers*





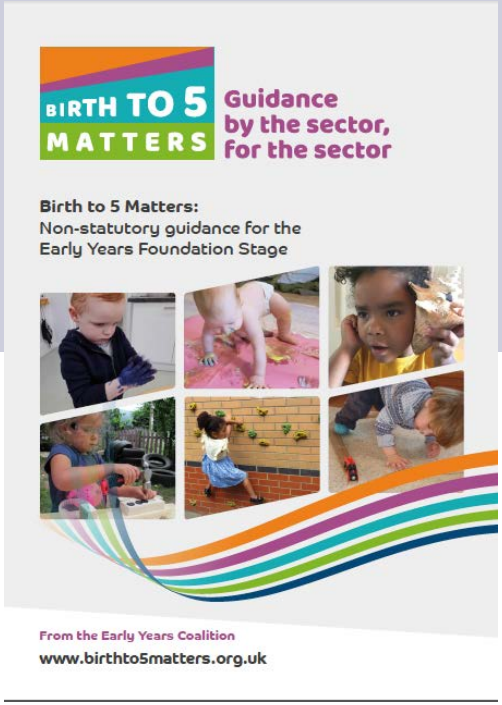


5 go in: one comes out



5 go in: 3 come out

what a child might be doing	what adults might do	what adults might provide
RANGE 6 	• Model comparing numbers in problems about fair shares. • Play games such as hide and seek that involve counting, forwards and backwards. • Talk with children about the strategies they have used to solve a problem. Spot opportunities to playfully pose composition problems for children to explore. • Discuss the order of numbers in context, e.g. finding a page number. • Enjoy subitising games and sustained shared thinking about number, indoors and outdoors. • Encourage cardinal counting by saying how many there are after counting (*...6, 7, 8. There are 8 balls*). • In everyday activities, ask children to count out a number of things from a group (e.g. *Could you get seven cups for snacktime?*) • Encourage children to make predictions and visualise the outcome in stories, rhymes and songs if one (or two) is added or taken away. • Talk to children about the marks and signs they use to represent and communicate their thinking. As appropriate, model and discuss informal and standard ways (e.g. using arrows, plus and minus signs). • Begin to model calculations in mathematical stories and number rhymes and in real contexts, using a range of ways of representing (e.g. five-frames). Use both informal and standard ways to record these, including tallies and symbols. Discuss children's own graphical strategies to solve problems, using some vocabulary of addition and subtraction.	• Involve children in voting, e.g. for books to read at story time, using linking cubes with children's names on. • Discuss examples and display large numbers including hundreds, thousands and a million. • Jump with children along a number track, counting each jump or counting on. • Sing counting songs and count together forwards and backwards, sometimes starting from different numbers and in different step sizes. Discuss numbers coming *before*, *after* and *between* and stress patterns. • Plan opportunities to order mixed-up numerals. • When counting groups as part of routines, e.g. self-registration with ten-frames, dinner chart etc,. record the final total as a label for children to see. • Subitise with children, talking about how they see numbers of things made up in a variety of arrangements (e.g. recognising odd and even numbers). • Pose everyday estimation problems and establish mental estimation benchmarks, e.g. more or less than 10. • Set up an estimation station where everyone records guesses; later count and order the guesses. • Build counting and ways of representing numbers into everyday routines. • Provide numeral cards for children to order on a washing line. • Play subitising games which involve quickly revealing and hiding numbers of objects, perhaps showing numeral cards and fingers. • Drop marbles into a tin and ask the children to listen (without looking) to count how many there are. • Provide opportunities for children to match a number of objects to the numeral, including zero, and display number lines to 100 at child height. • Provide dice, board and card games, sometimes involving older children, families and members of the local community. • Provide resources to make "staircase" patterns which show that the next counting number includes the previous number plus one. • Display children's mathematical representations, including explanations of the children's meaning
Comparison • Uses number names and symbols when comparing numbers, showing interest in large numbers • Estimates of numbers of things, showing understanding of relative size **Counting** • Enjoys reciting numbers from 0 to 10 (and beyond) and back from 10 to 0 • Increasingly confident at putting numerals in order 0 to 10 (ordinality) **Cardinality** • Engages in subitising numbers to four and maybe five • Counts out up to 10 objects from a larger group • Matches the numeral with a group of items to show how many there are (up to 10) **Composition** • Shows awareness that numbers are made up (composed) of smaller numbers, exploring partitioning in different ways with a wide range of objects • Begins to conceptually subitise larger numbers by subitising smaller groups within the number, e.g. sees six raisins on a plate as three and three • In practical activities, adds one and subtracts one with numbers to 10 • Begins to explore and work out mathematical problems, using signs and strategies of their own choice, including (when appropriate) standard numerals, tallies and "+" or "-"		



https://birthto5matters.org.uk/download-or-buy-a-copy/

A pedagogy for number sense: playful, practical and outdoors

- meaningful contexts
- routines –tidying up, snack time, register
- games: collecting, targets, hiding
- number rhymes
- picture books

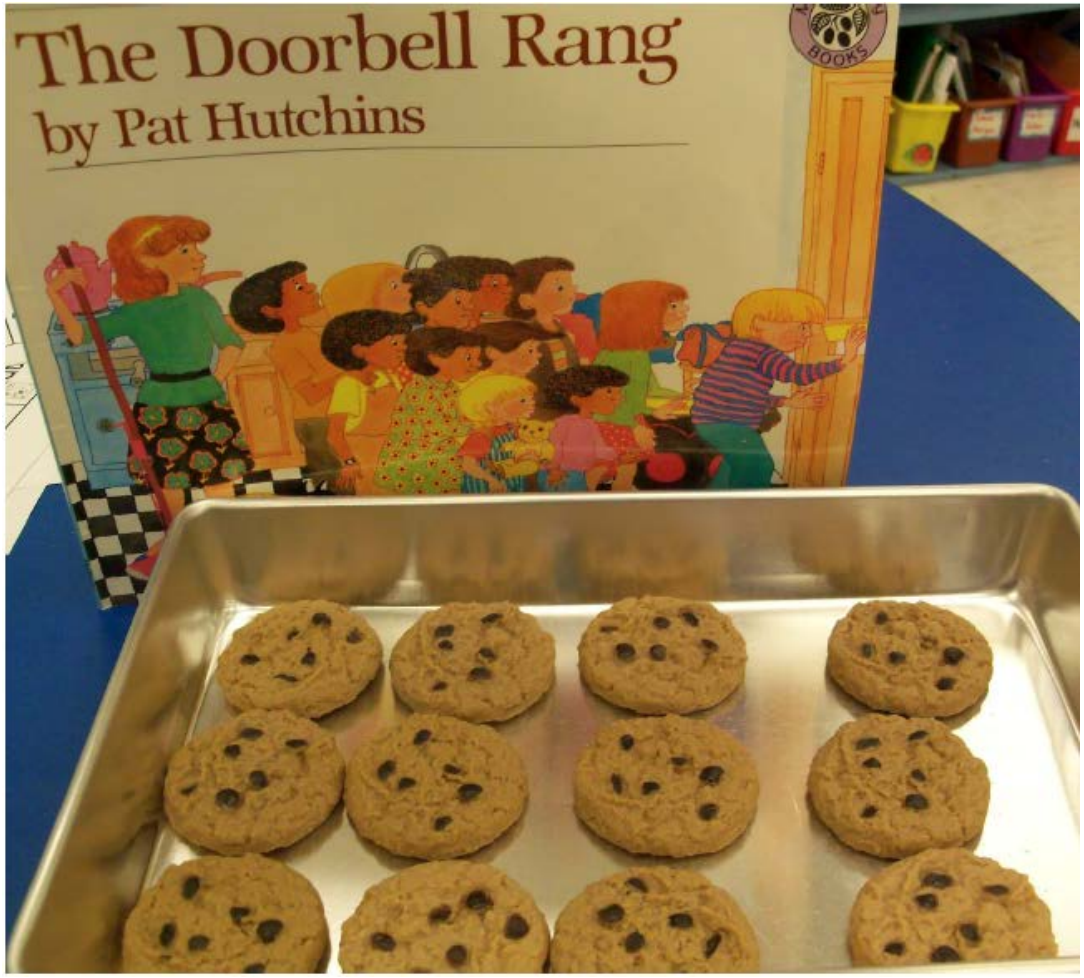
Number rhymes

One, two, three, four, five, once I caught a fish alive

Number rhymes support children's learning about numbers:

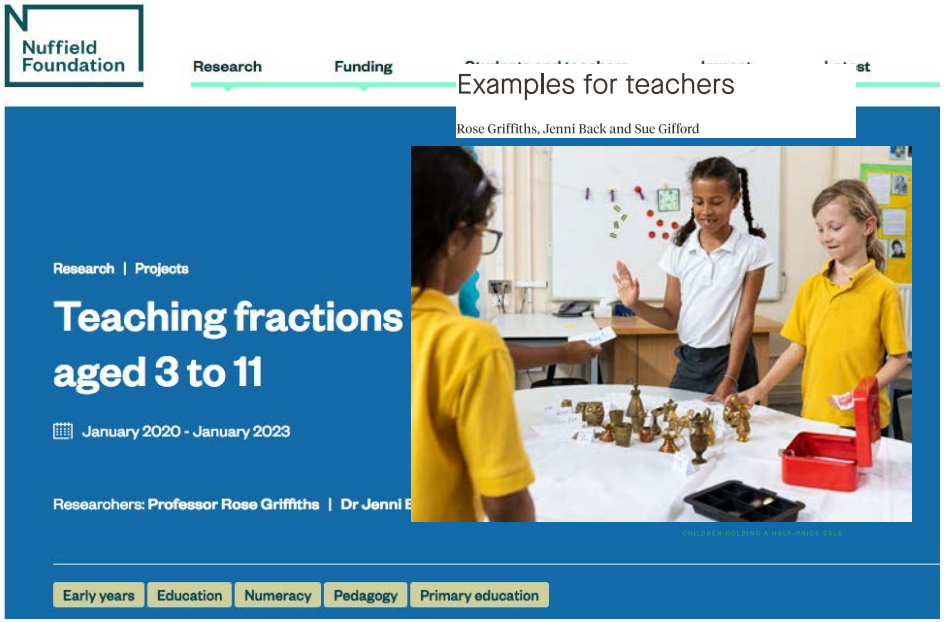
- music
- fingers
- numerals

Problem solving



Share the biscuits between two - then the doorbell rings…

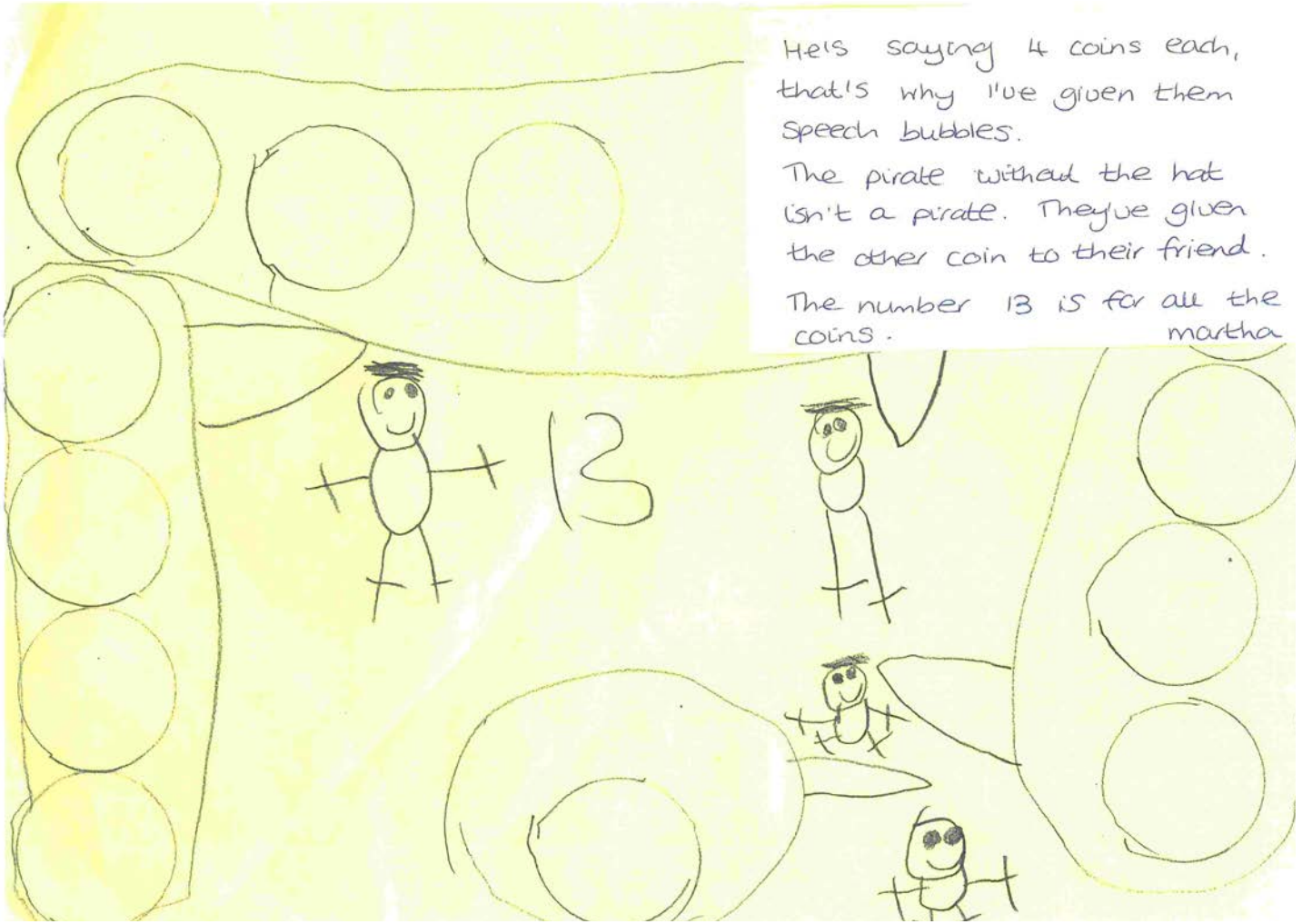
Sharing helps understanding of division and fractions



Vary problems:

- numbers
- remainders
- contexts
- objects

Children representing their thinking



Fiona O'Shea, Milton CoE Primary School

The characteristics of effective teaching and learning

In planning and guiding what children learn, practitioners must reflect on the different rates at which children are developing and adjust their practice appropriately. Three characteristics of effective teaching and learning are:

- **playing and exploring –** children investigate and experience things, and 'have a go'
- **active learning –** children concentrate and keep on trying if they encounter difficulties, and enjoy achievements
- **creating and thinking critically –** children have and develop their own ideas, make links between ideas, and develop strategies for doing things

Statutory framework for the EYFS

More than ever before, living and working in the 21st century requires the "four Cs" – creativity, critical thinking, communication and collaboration OECD 2016

Patterning

CHILD DEVELOPMENT

Child Development, xxxx 2016, Volume 00, Number 0, Pages 1–16

Early Math Trajectories: Low-Income Children's Mathematics Knowledge From Ages 4 to 11

We found that early patterning ..knowledge was a unique predictor of later mathematics achievement, over and above other mathematics and non-math skills.

Rittle –Johnson et al (2016, 2019)

Abstracting patterns is the basis of structural knowledge, the goal of mathematics learning

Warren 2005 (cited by M&M, 2009)

The beginnings of pattern awareness

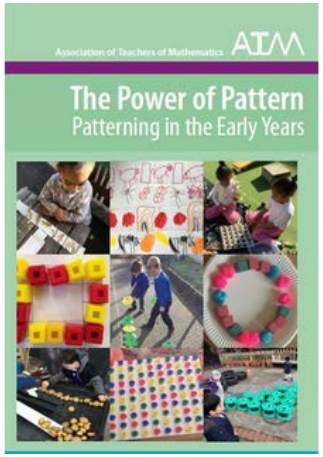
What makes it pattern?

Which bits are the same?

Reflecting, rotating, radial

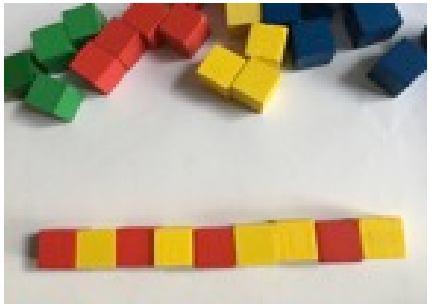
Growing - or shrinking

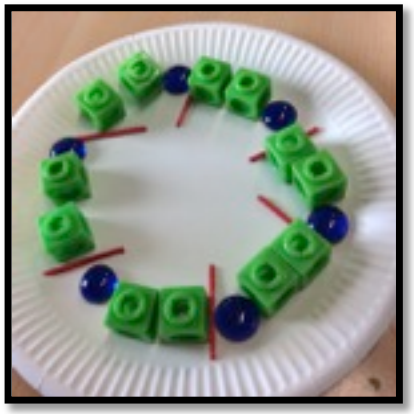
Repeating



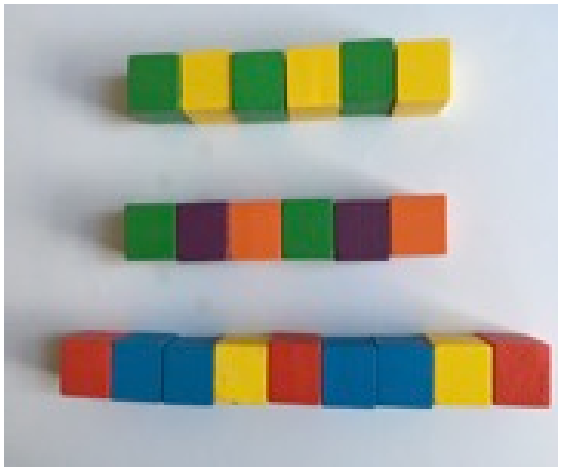
Progression in patterning:

- copy, continue, create, beyond AB
- fix errors
- **identify the unit**
- continuous patterns

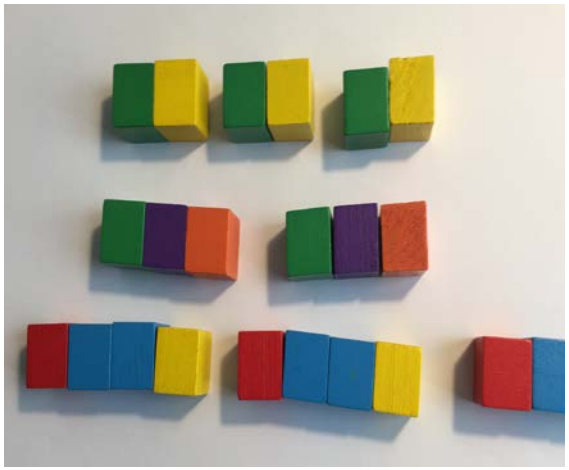


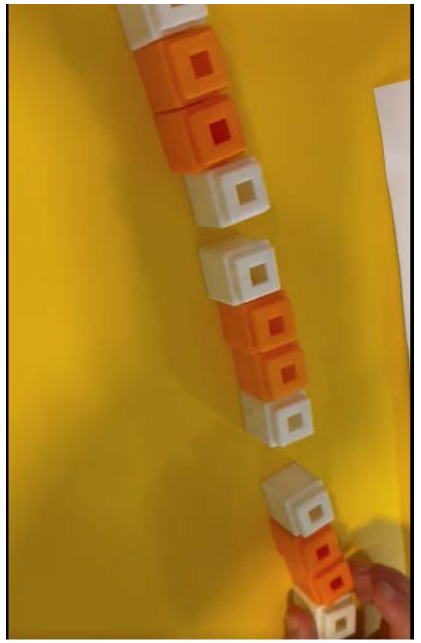


Identifying the pattern unit



AB, ABC, ABBC patterns





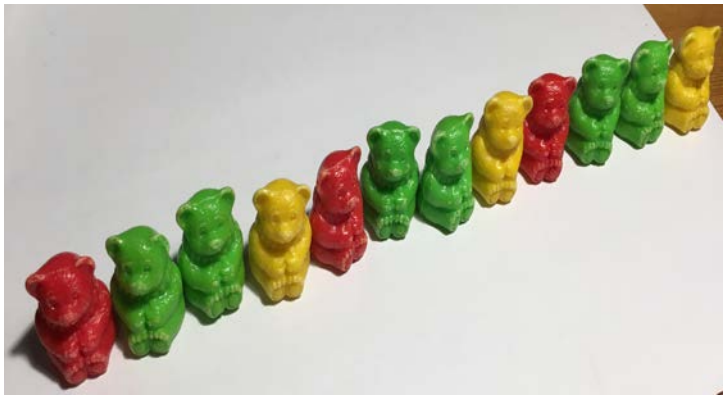
ABBA

Can you split these patterns into parts that are the same?

https://twitter.com/berniewestacott/status/1631668815661547520?s=20

Patterning assessments https://earlymaths.org/pattern-chips/

Symbolising and generalising



Sean made a pattern with bears.
Simon told him it was an ‘ABBC pattern’.
Sean: *So it could be dog, cat, cat, sheep?*

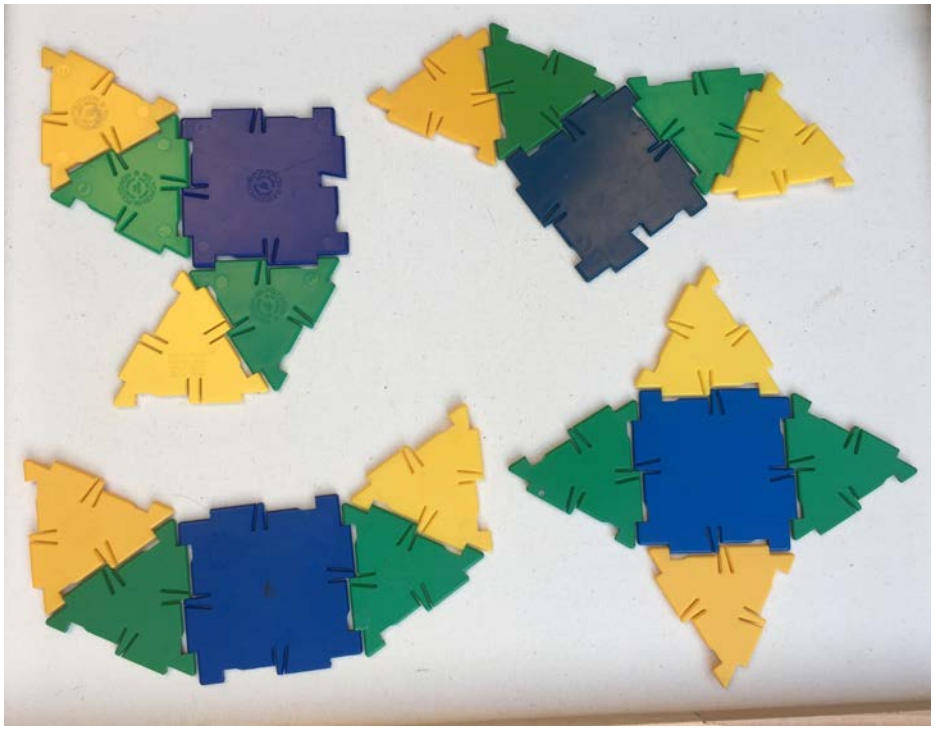
4 year olds using AB labels helps to generalise pattern structures

Fyfe et al, 2015

Symbolising action patterns

Spatial reasoning

Visualising spatial relationships



What does spatial reasoning involve?
Visualising and predicting

Shapes & properties
curved/straight, corners & sides
composing and decomposing

Spatial relations
positions & directions
transformations
navigation

Perspectives
models, maps, 3D / 2D

Spatial reasoning and careers

The relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related.

Hawes & Ansari, 2020

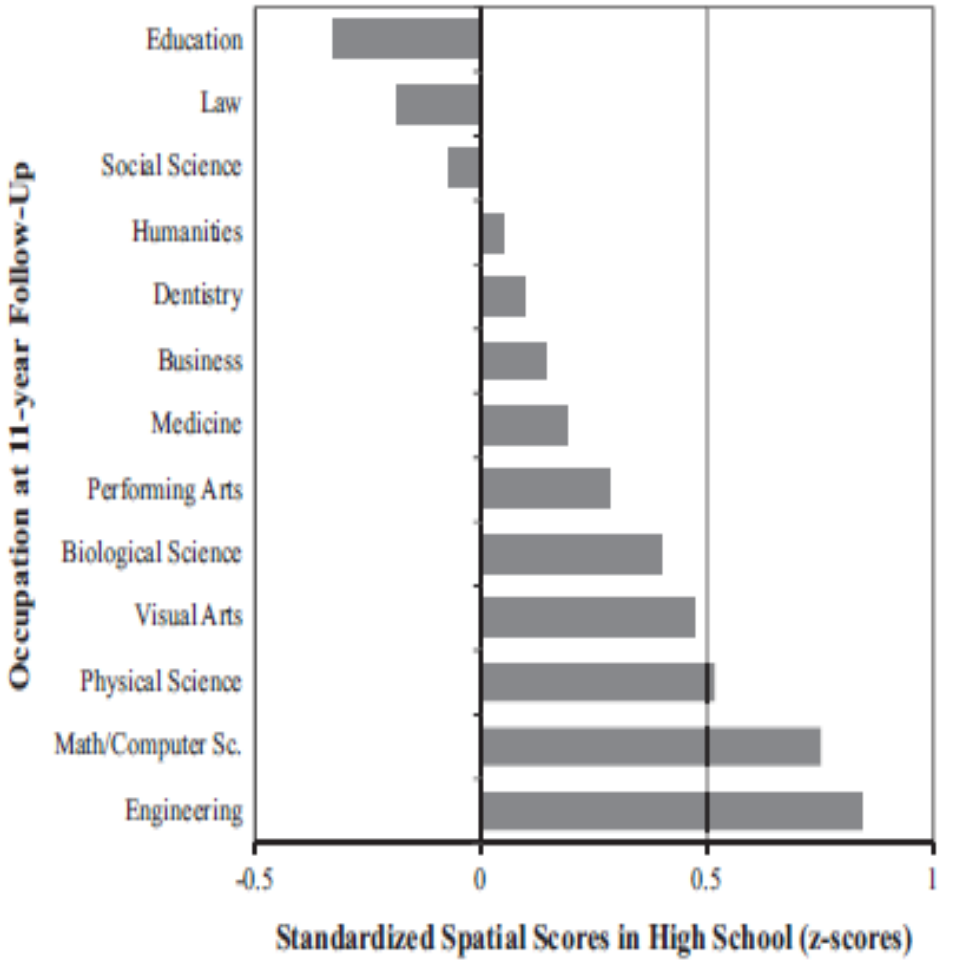
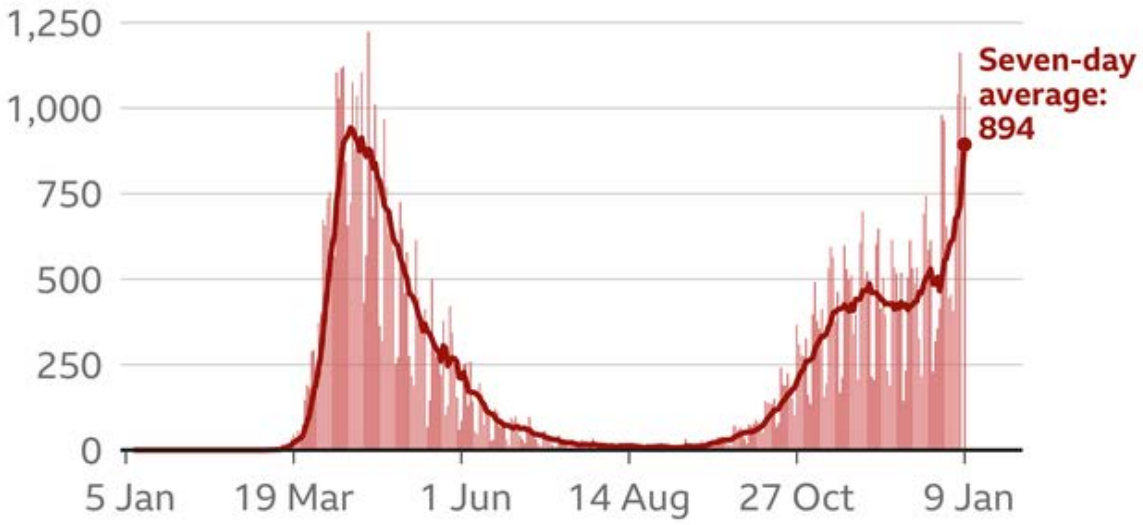


Fig. 1. Spatial Scores in 9th–12th Grade and Reported Occupations 11 Years Later.

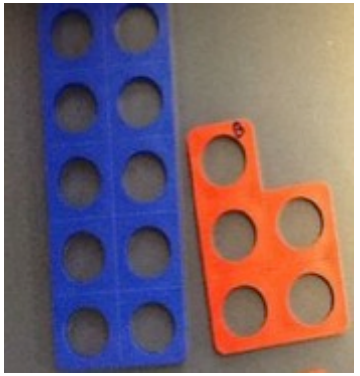
Verdine et al. 2017

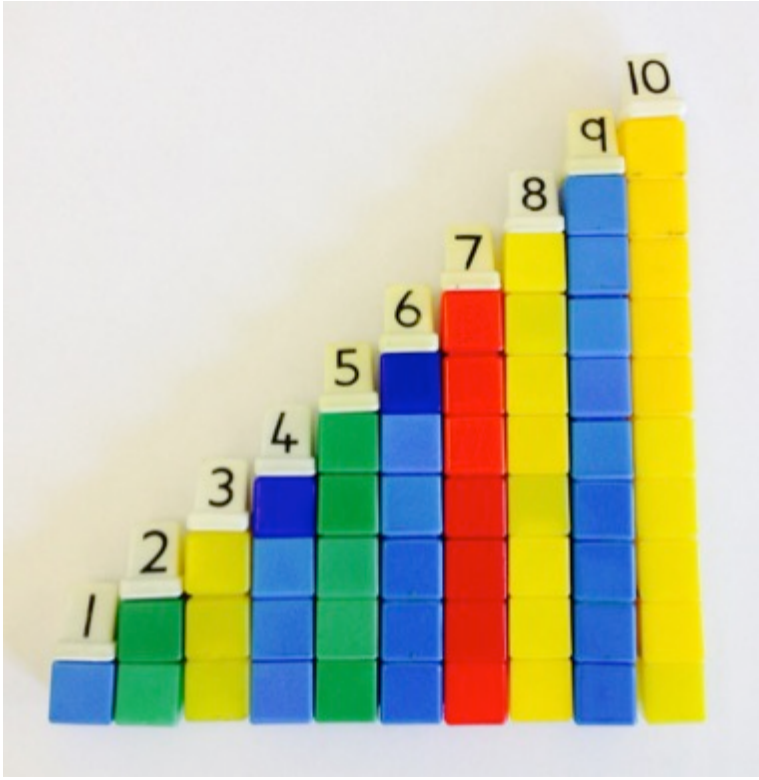
Spatial thinking plays a fundamental role in how people conceive, express, and perform mathematics. ..

Hawes & Ansari, 2020



Spatial representations help children understand number relationships





..Spatial thinking can be improved in people of all ages and through a wide assortment of training approaches. Hawes & Ansari, 2020

Optimizing spatial performance may be an underutilized route to improving mathematics achievement. Verdine et al (2017: 93,102)

Girls and other underrepresented groups are *harmed in their progression in mathematics due to lack of attention to spatial skills.* Sarama & Clements, 2009

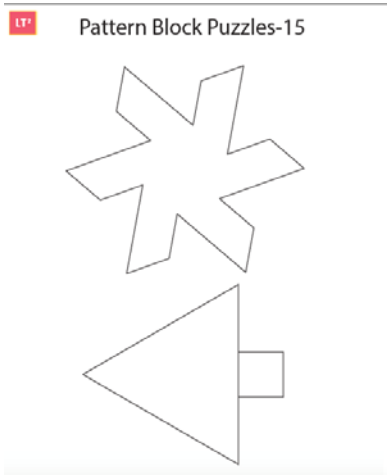
Which spatial skills predict maths?

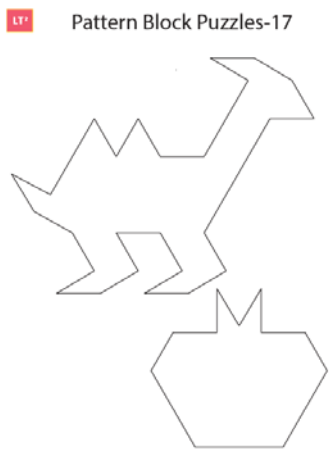
- Shape rotation Lauer & Laurenco, 2016
- Combining and positioning shapes Verdine et al, 2017

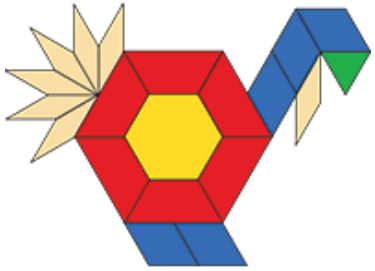
Puzzle play Levine et al, 2012

Construction

Wolfgang et al, 2003







https://www.learningtrajectories.org/pages/resources

Do we provide a progression in puzzles?

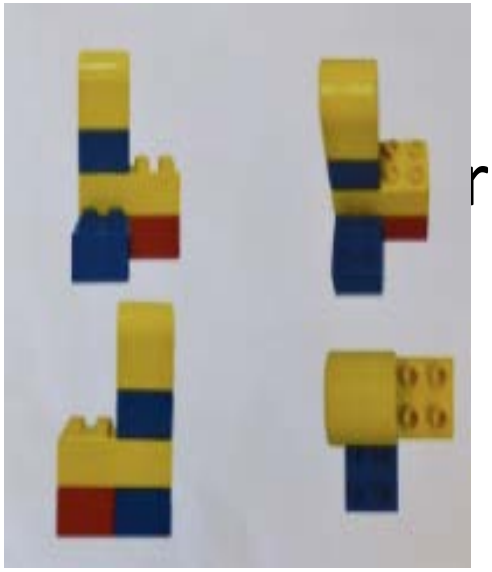
focusing on:
- shape properties,
- flipping and turning
- visualising

Alphabet jigsaw

https://earlymaths.org/mathematical-moments-3-4-5-year-olds/

Blockplay – development

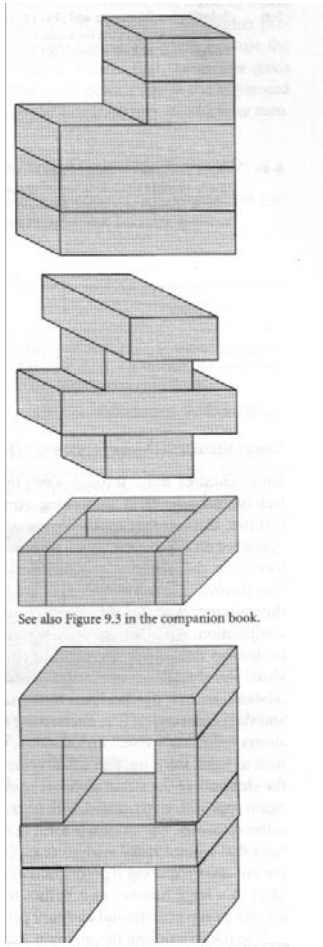
- stacking, assembling pieces
- composite shapes eg arches; trial and error
- **plans** composite shapes, towers of arches
- complex with repeated units; stairs
- units of units; ceilings Clements & Sarama (2009)

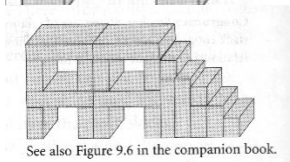


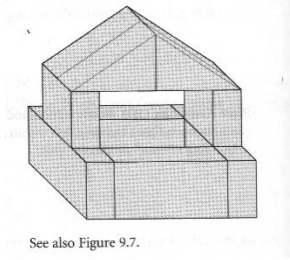
- story contexts

ructure challenges

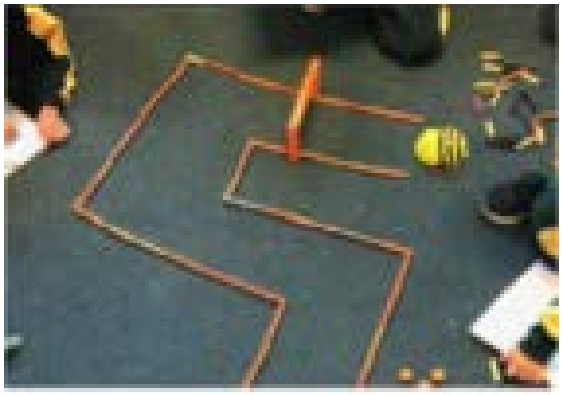
- making 3D models from 2D photos







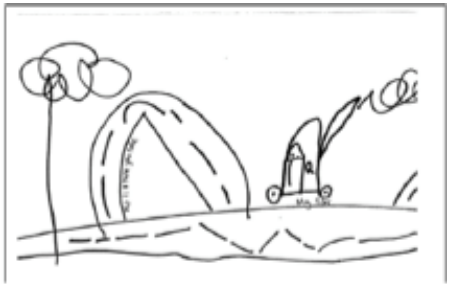
Spatial relations



Mulligan et al 2020

- obstacle courses
- small world play
- route finding
- treasure hunts
- robots
- models & maps

Sharon Palfreyman Corrie Nursery



Cohrssen & Pearn (2019)

How do we teach spatial reasoning?

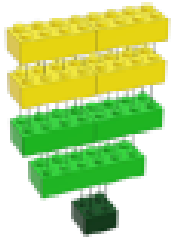
- body awareness
- spatial awareness
- large scale movement & toys
- language and gesture
- goal–oriented activities
- encouraging visualisation

More varied shapes

curved, straight, slanting, twisty, roof-shaped, star, corner, edge, right angle, concentric, spiral

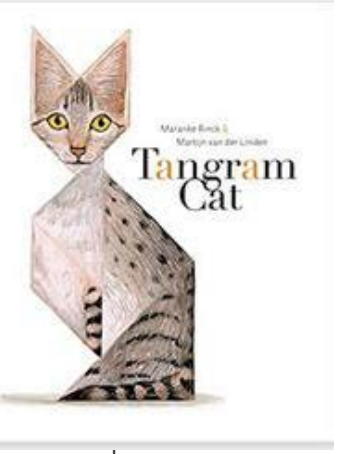
More specific vocabulary

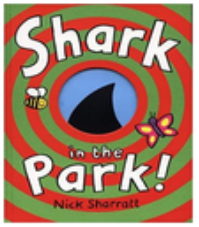
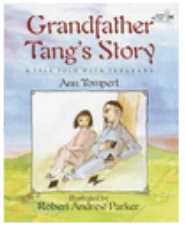
ECMG spatial reasoning toolkit

6 and 7 year olds	Children are learning to...	Adults might...	The environment might include...
	Interpret and predict what and how things will appear from different viewpoints **(perspective–taking)**, including when partially obscured or from above (plan view). Interpret and make **3D models and simple 2D maps** of familiar environments, identifying the representation of the real world feature. When drawing maps of familiar routes, **place things at approximately correct relative distances** e.g. near my home **Begin to use proportional language** e.g. *halfway, middle.* **Navigate simple routes**. Plan a simple route in a familiar environment using landmarks.	what it might look from the back or top. Show the actual model, view it from different perspectives and discuss how it looked the same or different in their head. Support children to build more complex constructions, using exploded model diagrams, e.g.  Encourage them to notice smaller units of combined shapes within models. Encourage children to create diagrammatic instructions, with drawing or writing, for others to make a model. Construct Lego marble mazes / roadways together, discuss left and right, forwards and backwards. Encourage problem solving. Create a classroom, school or playground map and give directions (referencing landmarks along the way) to find specific places or hidden items.	to make zig-zag folds and cut out people shapes holding hands (paper dolls). Mirror puzzle books such as '*The magic mirror book*' by Marion Walter. Resources and examples for making paper snowflakes: paper folded in half, then in three, to cut out designs on the fold. Images of constructions made with blocks (including exploded models) for children to discuss, compare and improve upon. Consider a 'Lego club' with family members or older children. Clipboards and pens for children to draw their models and design new ones. Plan views (or oblique views which are not quite above) of environments (e.g. classroom). Perhaps, use paper maps for role-play (e.g. travel agents) and Google maps for aerial photographs to identify familiar routes viewing them from above e.g. from school to the park or shops, from home to school. Plenty of opportunities to practise and develop confidence in playing bat and ball, over varying distances. Play at rolling balls down ramps and catching it, encourage children to invent their own anticipatory games.

ECMG Spatial booklist

https://earlymaths.org/spatial-books/



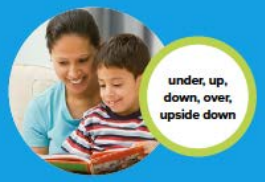
	Shark in the Park https://www.youtube.com/watch?v=24cU53mBKWY	Nick Sharratt	Perspective as Timothy Park 'sees' a shark's fin his telescope
	Snail Trail https://youtu.be/Q3e8rpJNZ1w	Ruth Brown	See the world through the eyes of Slimy Snail as he travels up, through, into, over and down.
	We're going on a picnic https://youtu.be/b5T99Bgl5vs	Pat Hutchins	Directions ('down the hill' and 'across the field') and the route they travel on their journey to have a picnic. Map making possibilities.
	Grandfather's Tang Story https://youtu.be/R7IvjeFyxYU	Ann Tompert	Story accompanied by rearranging tangram pieces to make the story animals.

ECMG **Spatial reasoning Toolkit** Posters

3-4 years
Spatial Reasoning Toolkit

At this age children are developing understanding of aspects of shape and space including composition (how shapes fit together), movements like turning and flipping, symmetry and scale. Children are beginning to recognise and predict familiar routes (e.g. to the park).

Puzzles
Moving, turning and predicting how pieces will fit



Books
Using spatial language

Obstacle courses
Experiencing and talking about directions

Small world play
Understanding position and direction

Out and about
Remembering and predicting routes, landmarks and directions, discussing perspectives and distance

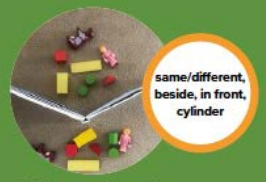
Pattern making and spotting
Arranging objects to make spatial patterns (position), noticing spatial patterns including symmetry in everyday objects

Block play
Using size and shape relationships as well as parts and whole to select blocks for specific purposes/ structures

https://doi.org/10.31234/osf.io/jnwpu
https://earlymaths.org/spatial-reasoning/
@EChildhoodMaths

4-5 years
Spatial Reasoning Toolkit

At this age children are learning to solve problems involving predictions and are beginning to use visualisation to imagine spatial information in the mind's eye (e.g. turning and flipping objects to see what will fit, mentally planning what to build). Compositions become more complex (e.g. combining shapes to make other shapes, reflections with four lines of symmetry). Children are developing their ability to follow and give directions and to use landmarks to find their way.



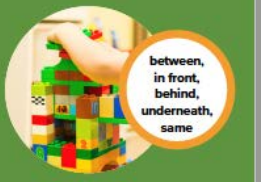
Hiding or barrier games
Developing visualisation, prediction and spatial language

Small world play
Exploring relative position, distances and transformation (turning and flipping objects)

Pattern making
Exploring symmetry (reflection)

Maps and models
Developing navigation and understanding of scale by using and creating simple maps and models

Puzzles
Understanding fit, composition and decomposition, through visualisation and discussion



Construction
Building constructions with arches and enclosures (perhaps linked to a story)

Books
Exploring shapes and sizes. Interpreting what book characters may see

https://doi.org/10.31234/osf.io/jnwpu
https://earlymaths.org/spatial-reasoning/
@EChildhoodMaths

Ofsted's definition of teaching - in both EIFs:

"Teaching is a broad term that covers the many different ways in which adults help young children learn. It includes their interactions with children during planned and child-initiated play and activities: communicating and modelling language; showing, explaining, demonstrating, exploring ideas; encouraging, questioning, recalling; providing a narrative for what they are doing; facilitating and setting challenges. It takes account of the equipment adults provide and the attention given to the physical environment, as well as the structure and routines of the day that establish expectations. Integral to teaching is how practitioners assess what children know, understand and can do, as well as taking account of their interests and dispositions to learn (characteristics of effective learning), and how practitioners use this information to plan children's next steps in learning and monitor their progress". Ofsted 2022: para 175, Ofsted 2022: para 393

The adult role

"Most of us probably have a good idea what it takes to get our young children to love reading. Snuggling up with a favourite book at bedtime, for example, sends a clear message about the value of reading.
***But what about a love of math*?"**
https://earlymath.erikson.edu

- **being playful**
  e.g. deliberate mistakes
- **sustained shared thinking**

Guided play (Skene et al, 2022)

- **a developmentally appropriate learning goal**
  (eg: using a learning trajectory)
- **child autonomy / control**
- **adult guidance**
  - providing sensitive hints/prompts
  - modeling
  - joining in the play [co-play]
  - adapting to the individual

Websites

Learning trajectories
https://www.learningtrajectories.org/

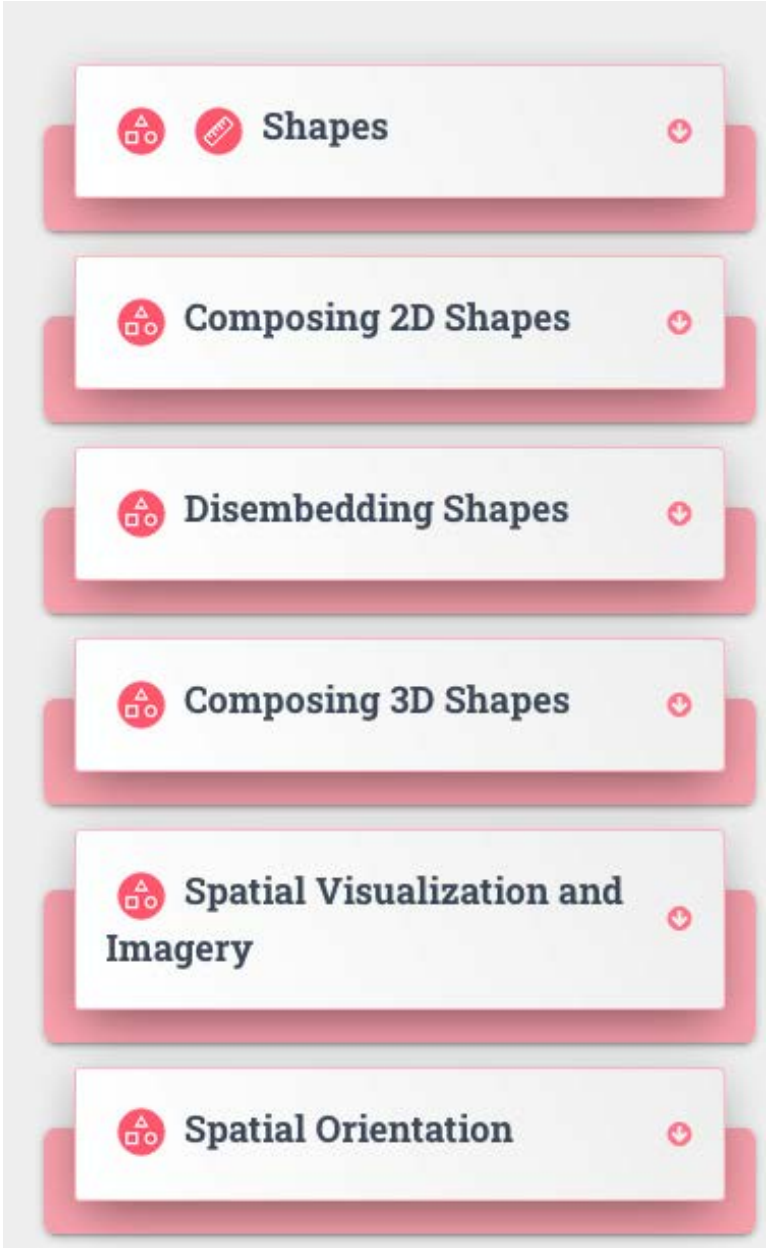
NCETM
https://www.ncetm.org.uk/resources/52505

Erikson
https://earlymath.erikson.edu/?s=Spatial+reasoning

DREME TE
https://dreme.stanford.edu/

NRICH
https://nrich.maths.org/13375

Early Childhood Maths Group
https://earlymaths.org/spatial-reasoning-toolkit/



References & links

Bower, C. et al. (2020) Piecing Together the Role of a Spatial Assembly Intervention in Preschoolers' Spatial and Mathematics Learning: Influences of Gesture, Spatial Language, and Socioeconomic Status. *Developmental Psychology*, **56**(4) 686–698

Cahoon, A., Gilmore, C. & Simms, V. (2021). Developmental pathways of early numerical skills during the preschool to school transition. *Learning and Instruction,* 75, 101484. https://doi.org/10.1016/j.learninstruc.2021.101484

Clements, D.H. and Sarama, J. (2021). 3rd Edition. *Learning and Teaching Early Math: The learning trajectories approach*. NY: Routledge

Cohrssen,C. & Pearn,C. (2019). Assessing preschool children's maps against the first four levels of the primary curriculum: lessons to learn. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-019-00298-7

Borthwick, A., Gifford, S. & Thouless, H. (2021) The power of pattern: *Patterning in the early years* Derby: Association of Teachers of Mathematicshttps://www.atm.org.uk/shop/All-Books/The-Power-of-Pattern---Patterning-in-the-Early-Years/ACT133

Davis, G. & Pepper, K. (1992) Mathematical problem solving by pre-school children. *Educational Studies in Mathematics*, 23:397-415.

Early Childhood Mathematics Group *Mathematical moments:* https://earlymaths.org/mathematical-moments-3-4-5-year-olds/

Early intervention Foundation (2018) *Key competencies in early cognitive development: Things, people, numbers and words.* Public Health England https://www.eif.org.uk/report/key-competencies-in-early-cognitive-development-things-people-numbers-and-words

Fyfe, E. R., Rittle-Johnson, B. & McNeil, N. M. (2015). Easy as ABCABD: Abstract Language Facilitates Performance on a Concrete patterning Task. *Child Development*, *86*(3), 927 – 935.

Giles, O.T., Shire, K.A., Hill, J.B., Mushtaq, F., Waterman, A., Holt, R.J., Culmer, P.R., Williams, J.H.G., Wilkie, R.M., & Mon-Williams, M. (2018) Hitting the target: mathematical attainment in children is related to interceptive-timing ability. *Psychological Science.*29(8) 1334–1345.

Gripton, C. & Pawluch, (2021) Counting collections in the early years. *Mathematics Teaching* 275 6-10

https://www.atm.org.uk/write/MediaUploads/Journals/MT275/02.pdf **Gunderson, E.A., Ramirez, G., Beilock, S.L., & Levine, S.C.** (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. Developmental Psychology, 8(5) 1229-1241https://doi.org/10.1037/a0028593

Hawes, Z. & Ansari, D. (2020). What explains the relationship between spatial and mathematical skills? A review of evidence from brain and behavior. *Psychonomic Bulletin & Review.* 27:465–482.

Lauer, J.E. & Lourenco, S.F. (2016) Spatial Processing in Infancy Predicts Both Spatial and Mathematical Aptitude in Childhood *Psychological Science* 27(10) 1291 –1298

Levine, S.C., Ratliff, K.R., Huttenlocher, J. & Cannon, J. (2012). Early puzzle play: A predictor of preschoolers; spatial transformation skill *Developmental Psychology*, 48(2) 530-542

Lowrie, T., Logan, T., Harris, D. & Hegarty, M. (2018). The impact of an intervention program on students' spatial reasoning: Student engagement through mathematics enhanced learning activities. *Cognitive Research: Principles and Implications* 3:50

Lyons, I.A., Price, G.R., Vaessen, A., Blomert, L. & Ansari, D. (2014) Numerical predictors of arithmetical success in grades 1-6. *Developmental Science* 17(5) 714-726 DOI: 10.1111/desc.12152

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, 21, 33–49.

Mulligan, J. et al.(2020) Evaluating the impact of a Spatial Reasoning Mathematics Program (SRMP) intervention in the primary school. *Mathematics Education Research Journal*, 32:285–305. https://doi.org/10.1007/s13394-020-00324-z

Nunes, T. & Bryant, P. (2009) *Key understandings in mathematics learning.* London: Nuffield Foundation.

Nunes, T., Bryant, P., Barros, R. & Sylva, K. (2012). The relative importance of two different mathematical abilities to mathematical achievement. *British Journal of Educational Psychology*. 82, 136–156.

OECD (2016) *How teachers teach and students learn: successful strategies for school OECD* Education Working Paper No. 130

Oudgenoeg-Paz,O., Leseman,P.P.M. & Volman,M.J.M. (2015). Exploration as a mediator of the relation between the attainment of motor milestones and the development of spatial cognition and spatial language. *Developmental Psychology, 51*(9), 1241–1253. http://dx.doi.org/10.1037/a0039572

Paliwal, V. & Baroody, A.J. (2018)How best to teach the cardinality principle? *Early Childhood Research Quarterly* 44 152-160

Paliwal, V. & Baroody, A.J. (2020) Cardinality principle understanding: the role of focusing on the subitizing ability *ZDM* 52 649–661 https://doi.org/10.1007/s11858-020-01150-0

Pruden, S.M., Levine, S.C., & Huttenlocher, J. (2011). Children's spatial thinking: Does talk about the spatial world matter? Developmental Science 14(6), 1417-1430. https://dx.doi.org/10.1111%2Fj.1467-7687.2011.01088.x

14(6), 1417-1430. https://dx.doi.org/10.1111%2Fj.1467-7687.2011.01088.x

Ribeiro,L.A., Casey, B., Dearing, E., Berg Nordahl, K., Aguiar, C. & Zachrisson, H. (2020) Early Maternal Spatial Support for Toddlers and Math Skills in Second Grade, *Journal of Cognition and Development*, 21:2, 282-311, DOI:10.1080/15248372.2020.1717494

Rittle-Johnson,B., Fyfe,E.R., Hofer, K.G., Farran, D.C. (2016) Early math trajectories: low income children's trajectory mathematics knowledge from ages 4 to 11, *Child Development* DOI: 10.1111/cdev.12662

Skene, K., O'Farrelly, C. M., Byrne, E. M., Kirby, N., Stevens, E. C., & Ramchandani, P. G. (2022). Can guidance during play enhance children's learning and development in educational contexts? A systematic review and meta-analysis. *Child Development*, 00, 1–19

Verdine, B.N., Golinkoff,R.M., Hirsh-Pasek, K, & Newcombe, N.S. (2017) "Links Between Spatial and Mathematical Skills Across the Preschool Years" *Monographs of the Society for Research in Child Development* http://onlinelibrary.wiley.com/doi/10.1111/mono.v82.1/issuetoc

Wolfgang,C., Stannard, L., & Jones, I. (2003) Advanced constructional play with LEGOs among preschoolers as a predictor of later school achievement in mathematics, *Early Child Development and Care,* 173:5, 467-475,

S.Gifford@roehampton.ac.uk